

Harrod's dynamics and the theory of growth: the story of a mistaken attribution

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Post-Keynesian growth theory is normally seen as originating from Harrod's 1939 'Essay in Dynamic Theory'. Harrod, however, was trying to lay the foundations of a new approach to economic dynamics, and often complained of misinterpretation. In this paper, the grounds of Harrod's argument are examined and compared with the 'textbook' interpretation. The latter is shown to be extremely reductive, as it ignores both Harrod's interest in the trade cycle and his methodological criticism of the 'time-lag theories of the cycle', and it also underrates the interesting implications of his non-linear approach and the epistemic implications of the instability principle.

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1. Introduction

Historical appendices to textbooks trace the origin of post-Keynesian growth theory to a 1939 article by Roy Harrod; his view of the process of advance is still sometimes referred to as a conception antagonistic to new (endogenous) growth theory. This interpretation of Harrod's dynamics as a theory of growth originated in the late 1940s, when Harrod recast his theory in a book (Harrod, 1948) and Domar stressed the similarity of Harrod's equation to his own. The 1950s and 1960s saw a proliferation of growth models based on Harrod's idea of the interaction of the multiplier and the accelerator, and in the 1970s the canonical interpretation was codified in textbooks on macrodynamics and growth.

While the recognition of Harrod's theory was consolidating, he repeatedly expressed his disagreement with dynamic modelling in terms of functional equations; he also claimed that, rather than providing a model of cycles or growth, he was laying the foundations of a new, revolutionary and more fundamental mode of approach to dynamic problems, and maintained that cycles are a necessary manifestation of growth phenomena. His disclaimers, however, were disregarded by commentators. The alternative (Frischian) notion of dynamics seems indeed capable of encompassing Harrod's and of solving at the same time other problems which he could barely formulate. It is therefore not surprising that

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Harrod's contemporary readers ignored his allegations of originality as to the *notion* of dynamics.

The interpretation of Harrod's dynamics as a representation of a long-period growth line seems to have been bequeathed to younger generations of scholars. The discrepancy between the story as told in textbooks and Harrod's own disclaimers does not seem to have attracted the attention of recent interpreters, who—with only a handful of exceptions—often take for granted that Harrod's object was growth theory.¹

If Harrod's disclaimers are to be taken seriously, it may well be that he was really trying to say something intrinsically different, and that textbook versions were more the product of the work of the exegetes than Harrod's own. This is what this paper aims to find out. Harrod's contributions to dynamics, from his inter-war writings to his 1973 book on *Economic Dynamics*, together with the mainstream post-war interpretations, will therefore be examined in the light of his qualifications. Sections 2 and 3 focus on Harrod's early writings. In Section 4, the main topics of the debates surrounding Harrod's dynamics will be presented while, in Section 5, I discuss of the limits of the textbook representation of Harrod's Dynamics. The following two sections are dedicated to Harrod's claim that he was providing the foundations of economic dynamics: his early writings are discussed first, then consistency with his later writings is examined. In Section 8, I conclude that Harrod's disclaimers were, after all, justified, and point out that the interpretations of his approach were reductive and failed to appreciate Harrod's point.²

2. The origin of Harrod's approach to cycles and growth

In later accounts, Harrod indicated that an essay on 'The expansion of credit in an advancing community', written in 1934, was his first contribution to economic dynamics,³ for it was 'an enquiry into the relation between the rates of increase in a regularly advancing society, with a view to determining what kind of system would allow the full potentialities of progress to be realised while being internally self-consistent' (Harrod, 1934B, p. 287). 'Dynamics' was not explicitly mentioned; in this *ante litteram* definition, however, we find indeed some elements which later characterised Harrod's notion of

¹ The titles of a number of recent well-informed and thorough writings focusing on Harrod's dynamics indicate how deeply this assumption is rooted in the economists' habit of thinking: "Anecdotal Myths": Tinbergen's Influence on Harrod's Growth Theory', 'Harrod's Growth Model and the Multiplier–Accelerator Model: Two Sides of the Same Coin?', *Harrod and his Trade Cycle Group. The Origins and Development of the Growth Research Programme*, 'Harrod on Harrod: The Evolution of a "Line of Steady Growth"', 'Economic Dynamics and the Theory of Steady Growth: An Historical Essay on Harrod's "Knife-Edge"' (Jolink, 1995; Coleman, 1991; Young, 1989; Asimakopulos, 1985, Kregel, 1985).

² It is not, of course, the first time that the textbook interpretation of Harrod is challenged, and two authors deserve to be cited in this connection. Asimakopulos (1985) emphasised that Harrod's notion of the 'warranted rate' and the instability principle cannot be pinned down to an immutable and rigid growth rate and to the knife-edge conception of neoclassical and neo-Keynesians exegetes alike. Nevertheless, Asimakopulos gives full weight to the 'line of steady advance', while barely mentioning the trade cycle. He thus fails to give the appropriate emphasis to the role in Harrod's trade cycle theory and in his notion itself of dynamics of both the instability principle and of changes in the warranted rate.

Kregel (1985), by focusing on Harrod's methodology, provides some extremely useful keys for interpreting Harrod's approach to dynamics. In particular, Kregel stresses that Harrod conceived of two distinct stages of dynamic analysis, an instantaneous one in which the equilibrium at one point of time is determined in terms of self-consistent growth rates, and a longer-period one in which parameters are allowed to change and which is therefore more appropriate to trade cycle analysis. Kregel rightly remarks that commentators, having failed to notice this distinction, tended to see Harrod only as a growth theorist who assumed that a unique growth rate exists. In spite of these helpful hints, the analytical roles of changes in the warranted rate are not made clear, nor the part played in Harrod's dynamics by the instability principle.

³ See for instance Harrod (1951, p. 261; 1952A, p. 221n; 1959, p. 451).

dynamics, although in his treatment there is a striking difference from the mechanism of unstable self-sustaining growth-generating cycles.

The similarity of this article to Harrod's later writings consists in the stress on the *regularity* of advance and on the *consistency* between the rates of growth. Having decided to disregard the trade cycle (Harrod, 1934B, pp. 287, 296), Harrod came to the conclusion that the growth of the system would be undisturbed provided the proportions between sectors were not altered. He was thus implicitly envisaging a system growing proportionately, and submitted that the right method for dealing with it is 'to take a cross-sectional view, assuming that the immediately preceding and succeeding periods yield similar developments, and to find out what assumptions with regard to the increase and mutual relations of the factors concerned are self-consistent and consistent with normal economic motives' (Harrod, 1934C, p. 478).

Harrod's first approach to dynamics, however, differed in a fundamental respect from his later contributions. The cause of advance, in fact, was taken as exogenously given, for it was due to the increase in productivity brought about by technological change. In *The Trade Cycle* (1936) and in his subsequent writings, on the contrary, growth originated from within the system itself: it was, in fact, one of the possible results of the interaction of the two fundamental principles driving the dynamics of the system, the multiplier and the accelerator.

The multiplier determines how much income increases as a consequence of an increase in investment, depending on the proportion of income saved; the accelerator determines how much additional capital is necessary to meet the expected increase in consumption, given the state of technology and the rate of interest. Each of these relations feeds on the other one: net investment generates additional income, which fosters expectations of an increase in consumption and thus new net investment. The outcome of this process depends on the 'dynamic determinants', that is, the coefficients determining the intensity of the multiplying and accelerating effects. Growth at a constant rate is one of the possibilities, and it would be an equilibrium state: if nothing changed, the system would continue to grow at the same pace. The occurrence and persistence of such a state, however, would depend on an appropriate combination of parameters.

Harrod stressed that nothing guarantees that the dynamic determinants are so well behaved. Their values continuously change: the propensity to save increases as income increases, while the distribution of income varies in the course of the cycle; technology changes exogenously, but the occasions to adopt the new productive techniques are more favourable at the bottom of the depression and the early stages of revival than at other times. The combination guaranteeing a steady advance can thus be reached and maintained only by blessed chance: growth, in fact, triggers the changes in the parameters that sooner or later undermine itself. Once the pace of growth diminishes, the fall is cumulative: less investment means less income, which implies a diminished prospected increase (or even a decrease) in consumption, leading to a further fall in investment.

The mechanism described by Harrod is thus intrinsically unstable. This is consistent with his view on the *possibility* of providing a satisfactory theoretical account of economic change. In his early writings, Harrod insistently pointed out that 'the right kind of explanation' of the cycle should include at the outset some destabilising factor, otherwise the system would settle into a state of rest.¹ The instability principle has thus an *epistemic* origin, and precedes the devising of the multiplier-accelerator mechanism in both logical

¹ Harrod to Haberler, 19 October and 5 November 1934; Harrod (1934A, pp. 465–70). For the role of this principle in *The Trade Cycle*, see Besomi (1997B, pp. 190–6).

and chronological terms. In *The Trade Cycle*, this principle was applied twice: as to statics (that is, the system of relations determining the *level* of output), Harrod identified a destabilising determinant (the general price level) capable of offsetting the tendency of the system to equilibrium and thereby enabling the system to undergo change. As to dynamics, the instability of the multiplier–accelerator mechanism¹ enabled him to account for the deviations from the equilibrium movement of the system (steady growth) and thus for the cycle. Eventually, the cumulative departure from equilibrium was checked both by changes in the dynamic determinants, and by some bottlenecks at the top or by the necessity of introducing replacements at the bottom of the cycle.

Harrod’s contemporaries understood an aspect which eventually escaped the attention of later readers, while they failed to appreciate what later became the *leitmotif* for interpretations of Harrod. On the one hand, most readers of *The Trade Cycle* recognised the role played by the non-linearity of Harrod’s approach: several reviewers of the book stressed that the end of a boom is brought about by changes in the value of the determinants.²

On the other hand, most commentators failed to realise that steady growth is an equilibrium state of the system, or they did not understand its role in Harrod’s trade cycle theory—in particular by dismissing this notion as hardly relevant to the cycle (Robinson, 1936, p. 692; Keynes to Harrod, 12 April 1937, in Keynes, 1973, p. 171). Harrod was disappointed by this lack of understanding, and soon re-elaborated his theory giving pride of place to growth rates.

3. The ‘Essay’

The first draft of the ‘Essay in dynamic theory’ (Harrod, 1939A, 1996) was ready by the beginning of August 1938, and was submitted to Marschak for comments and to Keynes for publication in the *Economic Journal*. Critical remarks and helpful advice came from both sides; Keynes’s criticism, in particular, induced Harrod to scrap the middle part of the old version, shifting the emphasis from the cycle to the ‘proof’ of instability. In the original plan, the article was neatly divided in three parts, corresponding to the methodological principles outlined in Harrod’s presidential address before the British Association (Harrod, 1938). The first, ‘dynamics proper’, was concerned with the determination of the actual and equilibrium (warranted) growth rates of the system at an instant, and with the instability of the warranted rate. At this stage, the parameters defining the intensity of the multiplying and accelerating effects were taken as given. The second part dealt with the ‘sequence of events’: Harrod illustrated the cyclical consequences of a divergence between actual and warranted rates, showing how the latter is dragged along with the former and overtakes it (thus determining an inversion of the movement, also cumulative in character) when the actual rate is halted by the ‘ceiling’ of full employment of work or resources (natural rate) or when investment falls below the level required for long-range capital outlay. At this stage, changes in the parameters were not only allowed, but were called upon to play an essential part in the argument. Finally, the third part dealt with the

¹ We shall see in the sequel that later formulations of Harrod’s mechanism did not automatically give rise to instability, in spite of his belief to have *proven* as a result what originally appeared as a *premise* of his reasoning.

² See for instance Benham (1938, p. 104), Smithies (1937, pp. 110–11), Anonymous (1937), Stafford (1937, pp. 74–5), Lokanathan (1938, p. 518), Gaitskell (1937, p. 475). A relevant exception was Tinbergen, who provided the first rendition of Harrod’s dynamic mechanism in terms of a linear functional equation (Tinbergen, 1937). Similar formulations became almost canonical after the war, until non-linear dynamics showed their intrinsic limits.

long period and policy consequences of the relative position of warranted, actual and natural growth rates.

Harrod's approach in the 'Essay' was essentially similar to his older theory.¹ First, the characteristics of equilibrium were examined: this consisted in growth at a constant rate which, given the right combination of parameters, would guarantee proportionate increases in investment and income justifying each other. Harrod's (1934) remarks on the appropriate method for studying a state of regular advance (see Section 2) apply here. Moreover, in 'Scope and method of economics', Harrod came to the conclusion that the first stage in scientific inquiries should be the drawing of a 'map' consisting in the simultaneous classification of the relationships between variables (Harrod, 1938; see, for a more detailed discussion, Besomi, 1996). These arguments together explain why, in the first nine sections of the 'Essay', Harrod concentrated on the determination of his growth rates *in one instant*.

Equilibrium, however, is unstable: a gap between warranted and actual rate increases, bringing (for instance) cumulative growth. But growth itself affects the warranted rate, since both the propensity to save and the capital coefficient change in the course of the cycle (Harrod, 1939A, pp. 29, 30). While the actual rate reaches a natural limit, nothing constrains the increase of the warranted rate, which sooner or later grows larger than the actual rate and sets in motion the reverse kind of cumulative movement. As was the case in the 1936 book, in the 'Essay' the cycle finds its '*vera causa*' (Harrod to Tinbergen, 1 July 1937, in Jolink, 1995, pp. 442) in the instability of equilibrium (although the specific cause could be any accidental event), the explosion being constrained within exogenous or semi-exogenous boundaries and the reversal being determined by endogenous changes in the values of the parameters.

When Harrod revised the 'Essay' under the pressure of Keynes's criticism, this part of the article was drastically compressed into two short sections.² The main line of argument is still clearly recognisable, but the reduced space certainly contributed to drive the attention of Harrod's readers away from it. Thus, while Marschak unequivocally understood that the first draft of the 'Essay' was concerned with the trade cycle and that the approach was essentially non-linear,³ both features were lost in post-war comments.

In this respect, the next version of Harrod's dynamic theory did not help much. In *Towards a Dynamic Economics* too, the trade cycle mechanism was only sketched. Harrod, however, claimed that he had outlined the essential elements of the theory, which only needed to be supplemented by the findings of other methods (Harrod, 1948, pp. 89, 91). In particular, he specified that the fundamental cause of the cycle is the cumulative character of a divergence between actual and warranted rates, while time-lags, errors and miscalculations play a secondary part in the argument (p. 89). It is interesting to remark that this passage explicitly draws the epistemic implications of the instability principle for the criticism of the 'time-lag theories of the cycle' and psychological explanations. In addition, Harrod stressed the non-linear character of his approach: ' G_w itself fluctuates in

¹ There were striking *analytical* differences between the two versions of Harrod's dynamics: the accelerator, for instance, was interpreted in *ex post* terms rather than as a *cause* of investment (see Besomi, 1995, pp. 318–24). Here, however, I am interested in pointing out the similarity in the *logical structure* of Harrod's arguments.

² See the Table of Concordances between the final version and the 1938 draft, published as an Appendix to the latter (Harrod, 1996, p. 279).

³ In his 'Remarks on R.F.H.'s "Essay in Dynamic Theory"', Marschak drew a diagram showing the oscillations of actual and warranted rates, the former being limited by the 'ceiling' (the diagram is reproduced in Young (1989, p. 167). It should be noted, however, that, in Marschak's original, in the phase of descending movement, the warranted rate line correctly crosses the actual rate line *below* the zero line).

the trade cycle', owing to variations in both proportion of income saved and capital coefficient (pp. 89–90). This is essential, for the turning points of the cycle are determined by the interplay of the natural rate of growth (and, symmetrically, of the replacements line) and fluctuations in the warranted rate. Curiously, however, this point was not noted. Instead, commentators cited a passage on p. 82, where Harrod wrote that he was *pro tem* assuming that the capital/income ratio is constant, failing to realise that this hypothesis was instrumental to the analysis of regular advance, and especially failing to take notice that, five lines below, it was stated that the warranted growth equation was soon going to be modified as a consequence of the production process becoming more roundabout (p. 83).

4. The post-war debates on growth models: filling in the missing piece, and the rigidity of Harrod's coefficients

At first the 'Essay' did not raise many comments and, during the war, Harrod himself was engaged on a rather different kind of work. It was 'rediscovered' after 1946, and attracted the interest of scholars, partly as a consequence of Domar having published two articles presenting an equation similar to Harrod's, and partly as a result of the publication of *Towards a Dynamic Economics*. Thereafter, the 'growth equation'—or the 'Harrod–Domar model', as it was soon renamed—was the subject of learned debates, prolonged until the early 1970s, when the resulting 'growth theory' was crystallised in textbooks.

The most controversial point was the instability result. The discussion took off from where it was left by Keynes and Marschak in private correspondence before the war. They had both pointed out to Harrod that his analysis of the stability of equilibrium was incomplete. This, in fact, would require the comparison between two different states of the system, while Harrod dealt with this matter in the first stage of the 'Essay', where he was concerned with an instantaneous picture (Keynes to Harrod, 29 August 1938, in Keynes, 1973, p. 334; Marschak, 1938). The same point was brought to the fore again after the war: Baumol couched the relationships of Harrod's 'Essay' in terms of a difference equation, and noticed that stability analysis was not possible unless a link between states was specified. He translated himself one of Harrod's statements into an additional equation describing the reaction of entrepreneurs when realising that the actual rate differs from the warranted one. He solved the system and concluded that Harrod's instability result was correct (Baumol, 1948). This approach opened a debate that lasted more than two decades. Other commentators, in fact, interpreted Harrod's statement differently (and somewhat freely), giving rise to a proliferation of 'Harrod–Domar models'.

The ambiguity of Harrod's notion of equilibrium contributed to increase the uncertainty as to the meaning of his statements. It was pointed out that six different definitions of the warranted rate were implicit in the 'Essay' (McCord Wright, 1949, p. 326), one of which stating that if the system grows at the warranted rate, then entrepreneurs (individually *and* as a body) are satisfied, and another one specifying that equilibrium advance would induce them to maintain the same rate of growth. This engendered additional confusion, for it was clear (as Harrod eventually had to admit) that maintenance of the same rate of advance was only *one* of the possible kinds of behaviour that satisfied entrepreneurs could adopt.¹

¹ Alexander (1950, p. 728), Harrod (1951, pp. 270–1); for a discussion, see Asimakopulos (1985, pp. 624–9).

Finally, in the late 1960s and early 1970s, reflecting the renewed interest in expectations, the missing link between states of the system was filled in in a different way, with entrepreneurs looking forward rather than over their shoulders. The system of equations was completed with explicit expectation functions, again differing from one author to another according to how Harrod's statements were interpreted.

Whether the stress was on the entrepreneurs' reaction to past experience or on their expectations, it is hardly surprising that the conclusions on the stability of equilibrium depended on the specific form of the function used to fill the gap left by Harrod. Some authors thus came to the conclusion that the system is stable, others that it is unstable,¹ and all missed the epistemic role the instability of equilibrium played in Harrod's theory of the cycle. Indeed, the cycle disappeared from view altogether. This was certainly helped by the fact that the continuous growth enjoyed by economies in post-war years diminished the urgency of the problems connected to depressions and raised the problem of explaining how the advance could be self-sustained. But the main problem lay in the kind of mathematical models in which Harrod's theory was represented.

Harrod's propositions on the multiplier, accelerator and equality of saving and investment were translated into a set of linear functional (mostly difference) equations, in which investment was interpreted as a function of the increase in income and saving as a function of the level of income; all variables were functions of time. Given an initial condition (the state of the system at some time), the solution of linear functional equations consist in the explicit forms of the unknown variables. This describes a path in time, which can take one of the following forms depending on the value of the parameters: exponential growth or asymptotic decline, exponentially amplified or damped fluctuations, or cycles of constant amplitude. The latter is a very special case, like the frictionless pendulum, because it corresponds to a particular value of the parameters only, thus making this kind of equation unsuitable for describing fluctuations. This language, given the assumption of linearity, cannot describe persistent cycles and growth at the same time, and thus fails to translate Harrod's original intention appropriately. It is suitable instead for describing steady growth, provided the appropriate combination of lags and values of the parameters are chosen.

The properties of this kind of equation thus forced modellers to concentrate on the first part of Harrod's article, the 'fundamental equation' describing the equilibrium rate of growth of the system, and to ignore the second, dealing with the cycle. Besides disregarding a theme which was dear to Harrod, this approach also overlooked at the outset an essential feature of Harrod's approach: the role played by changes in the parameters as income changes. To account for this, one should use non-linear equations, which can give rise to much more varied behaviour of the variables, including persistent cycles and growth at the same time. Economists began mastering the mathematics for dealing with these equations only much later (the pioneering work by Richard Goodwin dates back to the early 1950s, but the technique spread only with the diffusion of computing facilities), when the mainstream interpretation of Harrod's equation as describing a line of steady growth was already deeply rooted in economists' habits.²

The modellers' representation of Harrod's ideas was so successful that their assumption

¹ The literature is surveyed by Hahn and Matthews (1964), Miconi (1967), Pugno (1992, pp. 109–19 and ch. VI) and Pugno (1998). Harrod approved of Miconi's 'admirable catalogue of some of the misrepresentations' of his view: Harrod (1971, p. 78).

² Nonetheless, mathematical economists who recently took the trouble of reading Harrod's original writings have been able to provide more faithful interpretations of his work: see in particular Pugno (1992).

that coefficients are constant was soon attributed to Harrod himself, the evidence to the contrary notwithstanding. Since the early 1950s, neoclassical and neo-Keynesian economists alike ascribed to this hypothesis the cause of the instability of the warranted rate.

The idea that the economic system is not attracted towards a position of stable moving equilibrium runs counter to the presuppositions of neoclassical economics. Instead of taking part in the Harrod–Domar-model-building exercise and trying to find combinations of parameters that would guarantee stability, some of these theorists attacked the instability principle at its foundation. They diagnosed that the instability was due to a lack of flexibility in the original model, in particular in the capital/output ratio (Fellner, 1951, pp. 116–22). They thus introduced a production function, which was sensitive to changes in the rate of interest. This was meant to re-introduce the equilibrating mechanism, enabling the system to return to the warranted position—which was reinterpreted as a path of growth characterised by full employment of both capital and labour (Harrod’s notion of equilibrium did not refer to full employment, but only to the ‘satisfaction’ of entrepreneurs).¹

Neo-Keynesian growth theorists imputed the cause of instability rather to the rigidity of the propensity to save.² This criticism reflected the Cambridge interest in the distribution of income in relation to saving and accumulation. Harrod rightly protested that he had discussed these problems at length in *The Trade Cycle* (Harrod, 1970, pp. 737–8), and took them up again explicitly in his last book on dynamics (Harrod, 1973).

5. The textbook rendition of Harrod’s ‘Growth Model’

Mathematical modellers, neoclassical and neo-Keynesian economists, all failed to recognise the roles of instability of equilibrium and of the fluctuations in the value of the parameters: Harrod’s contribution was associated with Domar’s³ and was understood as providing a model of geometric growth. This reading was widespread and, throughout the 1970s, was crystallised in a number of textbooks on macroeconomics and on growth, on which the present generation of scholars was brought up.⁴ This interpretation of Harrod’s dynamics, however, does not fairly represent his view, either as it was originally expounded from 1934 to 1948, or with respect to Harrod’s comments on his commentators.

The textbook rendition of Harrod’s projects into the long period a formula originally thought of with reference to a single instant, and correspondingly prolongs the decision to ignore the trade cycle and the fluctuations in the value of the parameters. These problems

¹ Pilvin (1953), Yeager (1954), Solow (1956)—who caricatured the instability principle as the ‘knife-edge’—and Swan (1956).

² See in particular Kaldor (1951, pp. 843–4; 1957, p. 594n), Joan Robinson (1970, p. 732) and Pasinetti (1974, pp. 96–7).

³ The presumed similarity between Harrod’s and Domar’s formulas emphasises the similarities of the two approaches, but obscures the differences, which led to identify ‘their’ theories with a single equation (see Ahmad, 1961; Robinson, 1952, p. 47; Pilvin, 1952, p. 595); for a detailed account on the differences between Harrod and Domar see Asimakopulos (1986).

⁴ Some treatises on ‘formal dynamics’ appeared in the early 1950s: Baumol (1951) devoted a chapter to Harrod; Allen (1956, pp. 64–9) expounded the ‘Harrod–Domar model’. Soon after that, some treatises on growth discussed different models (Hamberg, 1956; Dusenberry, 1958). The first textbooks on macroeconomics devoting a chapter on growth appeared in the early 1960s (Ackley, 1961; Brooman, 1962), while since the end of the 1960s some volumes of ‘readings’ and the first textbooks on growth were made available (Stiglitz and Uzawa, 1969; Burmeister and Dobell, 1970; Sen, 1971; Hamberg, 1971; Wan, 1971; Neher, 1971; Kregel, 1972; Jones, 1975; Dixit, 1976; Hacche, 1979). However, the systematisation which eventually channelled most of the subsequent contributions was Hahn and Matthews’s celebrated 1964 survey on the theory of economic growth.

are linked together by their analytical role in Harrod's construction. Besides reflecting the methodological reasoning outlined above, Harrod's decision to constrain the analysis of the first part of his 'Essay' within an instant also mirrored a difficulty with his mathematics. Harrod did not know how to deal with the changes in the parameters intrinsic in his own formulation: he was aware of this problem, but his mathematics were clearly inadequate not only to solve it, but even to formulate it unambiguously. The instantaneous approach provided him with a useful short cut: within a time section of vanishing length, the changes triggered by the dynamics of the system do not have the possibility of cumulating their effects, and parameters can be taken as given. If one considers a longer period, this trick no longer works: the rigour of mathematics makes it necessary to recalculate, instant after instant, the new warranted rate starting from the new values of the parameters, or to integrate the variations of the coefficients within a non-linear equation. The first option implies calculating equipment, the second one mathematical capacities far beyond Harrod's reach (and his contemporaries', for that matter). Harrod could thus only tackle the problem in intuitive and qualitative terms. He was certainly aware of the limits of this approach: he recognised, in fact, that his treatment was somewhat tentative, involving 'a certain element of conjecture' (Harrod, 1996, p. 262). But he also rejected as too simplistic the attempts to reduce the fluctuations of the warranted rate to a regular sine-like curve: 'I dont think the graphical method especially well adapted to this subject [...] I find it hard to depict warranted growth since it may have various ups and downs within one cycle, being high in the early revival, for instance, because of surplus capacity, and again later on because of high saving' (Harrod to Marschak, 7 September 1938).

In the light of the division of Harrod's dynamics in different stages, with instantaneous 'dynamics proper' being prior to the analysis of the 'succession of events' for logical, methodological and analytical reasons, the extension to the long-run of the instantaneous rate of growth can hardly be attributed to Harrod himself.¹ Nor should it be forgotten that Harrod's intent was to give an outline of a theory of the cycle; but of this, there is only passing mention in the literature on Harrodian growth and macroeconomics.²

Moreover, Harrod's distinction between three different growth rates (the *actual*, the *warranted*—i.e., equilibrium—and the *natural*) disappeared from the textbook rendition of his dynamics.³ In Harrod's view, the comparison between actual and warranted rates was essential for studying the stability of equilibrium, the comparison between actual and natural determined the turning point in the cycle, while the comparison between natural and warranted revealed some long-period characteristics of the economic system. The textbook version was probably meant to bypass the confusion as to the role of these different rates which prevailed in the literature. For mathematical modellers, in fact, the

¹ Harrod had used some expressions such as 'line of output', 'line of advance', 'path of growth' and 'line of growth' in the 'Essay'. These, however, have to be considered in their context: Harrod used them to specify that the warranted growth was a *moving* equilibrium, and to distinguish it from the static equilibrium he was discussing in those pages (Harrod, 1939A, pp. 22–3).

² The non-linearity of Harrod's model and the interaction between growth and cycles was noted by students of non-linear models: Ichimura (1955, p. 25), Glombowski and Krüger (1982), Di Matteo (1987, p. 2), Punzo (1988, p. 31), Pugno (1992, pp. 121–8). Some noted that instability is an epistemic premise to trade cycle theorising before being an analytical premise for the operation of Harrod's trade cycle mechanism: Rose (1963, p. 72), Glombowski and Krüger (1982, p. 131) and Di Matteo (1987, p. 9n). The latter role was stressed by Hamberg (1956, p. 199), Kalecki (1962, p. 134), Brooman (1970 [1962], p. 346), Krauss (1970), Costa (1970, p. 403), Hacche (1979, p. 13), Nardozi (1983, pp. 141–4) and Asimakopulos (1986, pp. 287–8), and was implied parenthetically by Hahn (1971, p. xiii); Predetti provided a thorough discussion of Harrod's mechanism of 'cyclical instability' (1961, pp. 32–5, 87–94). Pugno (1992) is particularly important in this respect.

³ Kregel (1980) seems to be the only interpreter who appreciated Harrod's point.

warranted rate had no role to play, for the equations describe the actual path of the system without the need to refer to the equilibrium state. Moreover, neoclassical critics missed Harrod's point altogether, since they interpreted the stability problem as consisting in the comparison between natural and warranted rates (see, for a criticism, Hahn and Matthews, 1964, pp. 804–5; Sen, 1971, pp. 89–90).

A further shortcoming of the textbook interpretation of Harrod, the one which he resented most, consisted in treating the growth equation as a complete 'model' providing a full explanation of growth. Harrod had consistently insisted that he was only providing an 'outline of a theory' (Harrod, 1936, p. vii), he stressed the truistic character of the equation representing the actual rate of growth,¹ and that while 'so-called "complete models"' 'require special postulates and assumptions in regard to lags and coefficients, which can only be accepted subject to statistical verification', his own approach is based 'on assumptions of the utmost simplicity and generality' and 'makes no pretension to giving a complete explanation of the cycle'.²

6. Harrod's intent: the foundations of economic dynamics

The upshot of the previous section was that Harrod did not aim at elaborating a theory of long-run growth, he did not claim to have formulated a complete model, and was aware that his theory of the cycle was an 'outline' only. Nevertheless, Harrod believed he had something of importance to say. He consistently claimed, in fact, that he was proposing a 'revolutionary' way of thinking dynamic problems.³ The explicitness of Harrod's intent, however, was not matched by an equally clear and unambiguous notion of dynamics. In *The Trade Cycle*, which was Harrod's most detailed treatment of the subject, this notion was not expressly defined. However, several characterisations are scattered through the book, especially in opposition to statics, and it is therefore possible to reconstruct what he meant. Reviewers, unfortunately, were more concerned with the specific trade cycle mechanism Harrod had proposed than with his procedure and, accordingly, the novelty of Harrod's dynamic method was not recognised. In his subsequent writings on the subject, rather than providing a comprehensive view of the subject, Harrod stressed one or the other aspect of his concept of 'dynamics' according to the specific criticisms or misunderstandings raised by commentators. By that time, the definition of dynamics proposed by Ragnar Frisch and the 'econometricians' was almost unanimously accepted by economists working on cycles and growth, and Harrod's proposal must have sounded quite odd or even frankly obscure and irrelevant to them.

Harrod's characterisation of dynamics is rooted in the preliminary assumption that saving and investment entail growth. He thought the object and method of static analysis not to be suitable for such a case. Statics, in fact, enables one to determine the *level* of output corresponding to certain 'fundamental conditions' (i.e., costs and demand curves). These conditions of course change from time to time, and statics can deal with these changes by recalculating the equilibrium position corresponding to the new set-up. But in a progressive society, the fundamental conditions *continuously* change, while the method of statics can only deal with *per saltum* changes.⁴ Harrod aimed at developing a method of

¹ Harrod (1939A, p. 18; 1948, pp. 14, 77) and, for a more detailed discussion of truisms in connection to the notion of 'model', Harrod (1968, pp. 183–7).

² Harrod (1952A, p. 286. See also 1955, pp. 360–1; 1957B, p. 6; 1960, p. 277; 1969, p. 193).

³ Harrod (1936, pp. viii–ix; 1939A, p. 15; 1948, pp. 27, 80; 1953, pp. 553, 555; 1959, p. 451).

⁴ Harrod (1934C, p. 478; 1935, p. 727; 1936, pp. 166–7); Harrod to Keynes, 6 and 7 April 1937, in Keynes (1973, pp. 163, 166); Harrod (1939A, p. 15; 1948, pp. 7–8; 1957A, p. 193; 1973, p. 3).

analysis analogous to statics, but capable of accounting for, and dealing with, the changes in the fundamental conditions triggered by advance itself.¹ He thus looked for the characteristics of equilibrium movement which consisted in steady growth—'steady' having the double meaning of 'continuous' and 'at a constant pace'. Correspondingly, in his early writings he defined the object of dynamics as being the *rate of change* of output (Harrod, 1936, pp. 89–90, viii). Later, having devised the equation describing the rate of growth of income, he defined dynamics 'as referring to propositions in which a rate of growth appears as an unknown variable' (Harrod, 1939A, p. 17); however, he continuously stressed his original intuition (dating back to 1934, *before* he devised the multiplier–accelerator mechanism) that dynamic equilibrium is characterised by the mutual consistency of rates of change at a given point of time (1960, p. 277; 1962, p. 1009; 1963, pp. 402, 403).

Harrod's notion of dynamics thus presents several facets: the *methodological* distinction between continuous and discontinuous change and the stress on instantaneous analysis, the *ontological* characterisation of the object of analysis for the presence or absence of saving and investment, the *analytical* distinction of level and rate of growth of output, and the *epistemic* postulate of instability. Taken together, they reveal that Harrod had in mind an organic, and somewhat grandiose, scheme of thought. This was not free of drawbacks: besides those listed above (in particular the ambiguity of the concept of equilibrium, Harrod's inability to deal with variations in the parameters and to develop his theory of the cycle with more precision), one should mention his failure to link static and dynamic laws (see Besomi, 1997B), the fallacious classification of the analysis of stability as belonging to the first stage of dynamics, while it requires instead a comparison between successive states of the system, and finally the failure of his equations to represent the causal direction in the relationships between variables as expressed in Harrod's axioms (see Besomi, 1995, pp. 322–4). Yet, in spite of these limits, Harrod had some powerful ideas deserving to be taken seriously. Firstly, his (verbal) non-linear approach enabled him to encompass growth *and* cycles theoretically as specific aspects of the dynamics of the economic system. Although Harrod did not master the mathematics for developing his intuition, he envisaged a general movement much more complex than his contemporaries could even imagine. Secondly, his instability principle gives expression to the correct idea that a proper explanation of permanent fluctuations must rely on forces originating from within the system itself, rather than on exogenous causes. The fruitfulness of this principle is witnessed by the fact that it also implicitly emerges from non-linear models of the cycle: fluctuations, in fact, can be represented on phase diagrams by stable closed lines (limit cycle) or limited areas (strange attractors) surrounding an unstable stationary point. This translates the idea that the cycle results from the interplay of forces keeping the system away from equilibrium and other forces preventing the system from disintegrating. This is precisely what Harrod was trying to say without, however, being understood until the advent of non-linear dynamics.

Having failed to attract attention to his notion of dynamics with *The Trade Cycle*, Harrod assumed a somewhat defensive attitude, replying to specific criticisms rather than expounding in detail his line of thought. He thus stressed one or the other of the peculiarities of his dynamics, but failed to make his overall point understood. His readers were more interested in the properties of the 'model', and Harrod's claims of having proposed a new and revolutionary mode of thought could not impress them. Taken one by one, the

¹ On the methodological continuity between Harrod's dynamics and 'traditional theory' (i.e., the partial equilibrium approach as thought in Cambridge in the 1920s), see Besomi (1997A).

features of Harrod's dynamics looked like a subset of, or a strong limitation upon, the properties of the econometricians' notion of dynamics. The solution of a functional equation gives both the level and the rate of growth of the variables, at one instant and at all other points in time, allowing for continuous changes in the variables, independently of whether or not investment is going on. The stability of equilibrium is a result, to be proven or disproved, depending on the form of the equations describing the economic system. Harrod's claims on his 'way of thinking dynamic problems' must therefore have appeared rather trivial in the eyes of mathematical modellers. Nonetheless, they found Harrod's multiplier–accelerator mechanism quite useful in its own right, they explored its properties, and argued about the only debatable point left, namely, the stability of equilibrium. Hicks was the only reader of Harrod who commented on his notion of dynamics—probably thanks to the fact that he himself had proposed a definition alternative to the dominant one—rightly pointing out that

To anyone who comes to the Harrod theory from the theory of the econometricists, it looks at first sight to be nothing more than a weaker, and less usable, version of the 'macrodynamic' model. What Mr. Harrod seems to be doing, if one comes to his theory from that side, is simply to be elaborating one of the cases which, on the other line, had been passed by and rejected in favour of a more interesting alternative. (Hicks, 1950, p. 7)

Harrod, however, had a grievance against the econometricians' method of analysis.¹ He interpreted their lags as representing delays and frictions in the system's adjustment to equilibrium, and saw these approaches as equivalent to the theory of the cycle which he was criticising in 1934 and which prompted him to develop his epistemic principle of instability. In Harrod's view, these theories were based on a view of the economic system where a stable equilibrium would prevail, if it were not for some difficulties in the process of adjustment preventing the settling into a state of permanent rest.

To this view, Harrod opposed his instability principle, which he believed to provide a more satisfactory 'kind of explanation': if the system is intrinsically unstable, one does not need to rely on maladjustments or external *ad hoc* causes of fluctuations (Pigou's optimism and pessimism, for instance: Harrod, 1934A, pp. 469–70) which generate instead from the nature of the system itself. The instability principle is therefore the *vera causa* of the cycle, while lags play a relatively minor part (Harrod to Tinbergen, 1 July 1937, in Jolink, 1995, pp. 441–2). Harrod's insistence that his explanation was more 'fundamental' than the rival ones, even when the econometricians' notion of dynamics was generally accepted and his 'model' was interpreted in those terms, reveals that he consistently *believed* in the instability of the warranted rate on *epistemic* rather than *analytical* grounds, in spite of Keynes's pressure to 'prove' this result in 1938 (see Section 3) and despite the post-war debates on the stability of the 'growth path' resulting from Harrod's 'model'.

7. Harrod's last writings: re-laying foundations

In his post-war writings, Harrod often resumed the thread of his original formulation and restated the cornerstones of his theory, although he also introduced some shifts of

¹ Harrod's critical targets were all the 'time-lag theories of the cycle'. This notion included at first Robertson's period analysis (Harrod, 1936), the Swedes' sequence analysis (Harrod, 1937, pp. 496–7), Hicks's definition of dynamics as referring to variables which must be dated (Harrod, 1939B; 1955, p. 360; 1963, p. 403; 1973, p. 16). After the war, Harrod focused on Tinbergen's and the econometricians' lagged (or integral) functional equations.

emphasis. The continuity with his pre-war writings lies in the repeated claim of working towards the foundations of a dynamic theory, in the stress on the division of dynamics in distinct stages and in the recognition of the instability principle as the primary cause of the cycle. Harrod, however, like to his contemporaries, became more interested in the process of advance than in the trade cycle, as the latter problem was made less urgent by the evolution of an economic situation towards a trend of apparently uninterrupted prosperity.

The essence of Harrod's main claim was not altered in the course of the years. In 1972, when he re-wrote *Towards a Dynamic Economics*, Harrod felt—as he had at the end of the war—that ‘we are still only on the threshold of the subject’ and that the resulting book, *Economic Dynamics*, ‘should be regarded as a first beginning’ (Harrod, 1973, p. vii; see also 1963, p. 403; 1971, p. 78). Dynamics was still defined as being ‘concerned with the determinants of the rates of increase of the main categories of demand’ (Harrod, 1973, p. 11), and as to the method of inquiry, Harrod insisted on the logical priority of uniform advance over inflections and discontinuities and of instantaneous analysis over time-lags (p. 16). This statement recasts the division of dynamics into stages advanced in 1938, and was even formulated by means of the analogy of a train in motion at a constant velocity, which he had first used for expressing the dynamic problem in 1934 (Harrod, 1934C, p. 478; 1974, p. 16). Harrod also insisted on this specific point in a number of his intermediate writings. In particular, he had stressed that ‘as a matter of methodological expediency, the analysis of regular advance should precede, if possible, that of irregular advance. Knowledge about the necessary mutual relations between variables, exogenous or dependent, in a regular advance ought to throw light on observed relations in an irregular advance.’ (Harrod, 1963, pp. 402–3). He drew two conclusions. The first was that ‘if we first concentrate on necessary relations in a regular advance we need and should consider *one point of time* only’ (*ibid.*, p. 403; see also 1959, p. 454; 1960, p. 279; 1962, p. 1009; 1973, p. 16). The second conclusion regarded the notion of dynamics, which was accordingly defined as concerning the mutual relations between the rates of increase (or decrease) of certain magnitudes in a growing economy (1955, p. 359; 1960, p. 277; 1962, p. 1009; 1963, p. 403).

The logical place of the trade cycle in this scheme of thought remains the same as in 1939, as it consists in deviations from the equilibrium rate of advance to be examined *after* the fundamental propositions have been established. The explanation of the phenomenon also remained essentially unaltered, for Harrod consistently indicated its cause in the instability of the moving equilibrium (see for instance 1951, p. 262; 1973, pp. 40–2, 45; 1976, p. 72) and characterised its course in terms of the changes in the value of the parameters (1959, p. 454; 1970, pp. 737–8; 1973, pp. 36–42). Although, after decades of debate on the stability of equilibrium, the epistemic role of the instability principle was pushed into the background, Harrod occasionally drew out some of its implications. For instance, commenting on Tinbergen's failure to find statistical evidence regarding the action of the accelerator, Harrod pointed out that one should look instead for the *failure* of investment to adapt to the increase in demand as a cause of the cycle: ‘if investment corresponded to requirements at each point of the cycle, there would be equilibrium at each point and no cause of further movement upwards or downwards’ (Harrod, 1952B, p. 133). Or, again, he restated his interpretation of the psychological theories of the cycle as being founded on the amplification of accidental deviations from equilibrium by ‘an independent basic cause (the instability principle)’ (1973, pp. 41–2).

Although the trade cycle retained its place as the second stage of dynamics, and notwithstanding his remark that ‘integration of growth theory with trade cycle theory is a

virtue' (Harrod, 1957B, p. 6), after his 'Notes on trade cycle theory' (Harrod, 1951) this subject was not given much space in Harrod's writings. Moreover, as a reaction to the neoclassical interpretation, Harrod played down the degree of instability of the system, suggesting that the metaphor of the knife-edge be substituted by that of a shallow dome (Harrod, 1970, p. 740) or of a grassy slope (Harrod, 1973, p. 32).

In his last book on dynamics, however, Harrod resumed his thread of thought on the cycle (Harrod, 1973, pp. 36–41). The mechanism he devised is essentially similar to that briefly expounded in 1939. However, Harrod resumed from the 1938 draft the distinction between a 'normal' warranted rate, pertaining in a steady advance, from the special values it attains in the course of the cycle.¹ This characterisation has two merits: it emphasises the non-linear nature of Harrod's approach, and clarifies the idea that the warranted rate does not itself represent an equilibrium value: it would only do so on condition of having coincided for some time with the actual rate, giving rise to steady growth (see also Harrod, 1973, pp. 20, 31).

The distinction between normal and special warranted rates helps place the relationship between dynamics, trade cycle and growth theory in the right perspective. Harrod's main concern was economic dynamics, more as a *concept* than for its *analytical results*. He continually emphasises in his book—consistently with his previous attitude—that he is still laying foundations only, formulating the 'axioms that would be basic in a general theory of economic dynamics' in the form of three equations describing the actual, warranted and natural growth rates (*ibid.*, p. 167). Growth economics 'has a wider connotation than dynamic economics' (p. 1): it includes sociological considerations, and one must be aware of the 'danger that these wider interests may deflect attention from the fact that dynamic economics in the narrow sense urgently needs its set of basic axioms, in which, so far, it has been rather lacking' (p. 2). As to the trade cycle, Harrod stressed that it 'is one aspect of the growth process' (p. 41). Its proper place is logically subsequent to the analysis of the mutual relationships during regular advance (p. 16), and provides the analytical instruments for analysing the implications of monetary or fiscal policies on the growth rates of the system (ch. 7).

8. Conclusion

In the course of time, Harrod introduced a number of changes in his dynamic theory, particularly regarding the interpretation of equilibrium (following the criticisms of ambiguity), the function of the accelerator, the role of interest (in response to neoclassical criticism), and the shift of emphasis from cycles to advance. However, some fundamental features remained unchanged: the dynamics of the system results from the interaction of the multiplier and the accelerator, the possible equilibrium movement (steady advance) is unstable, thus giving rise to the cycle, and the relationships are highly non-linear. The method of inquiry also remained the same, as Harrod consistently stressed that the mutual relationships between the rates of changes of magnitudes in a state of steady (that is, constant *and* continuous) progress at one point in time should be studied first, while only at later stages is it possible to study irregular modes of advance (such as the trade cycle) in their unfolding in time and thus provide policy advice.

These changes and continuities should be seen in the perspective of Harrod's goal. He did not aim at elaborating refined 'models' of growth or cycles, but at devising 'tools for

¹ Harrod (1973, p. 36; 1996, pp. 264–5). For a discussion of the differences between the two versions of the 'Essay' see Besomi (1996, pp. 284–7).

thought' permitting a theoretical framing of these phenomena. In spite of Harrod's resolute insistence on the epistemic and methodological aspects of his dynamics, commentators ignored his claims, and interpreted his contribution as a theory explaining the causes and modes of long-period growth, caricaturing it as the 'Harrod-Domar model'. This misunderstanding quickly gained acceptance among economists, and was crystallised in textbooks on growth theory and macrodynamics. If this recognised the novelty and importance of Harrod's pet idea of the interaction of multiplier and accelerator, it also contributed to the oblivion into which the purpose of a lifework was condemned in favour of the success of the econometricians' competing notion of dynamics. It is therefore an irony of history that the academic success of Harrod's dynamics was rooted in the eponymous linkage with a 'theory' he did not father and to an interpretation of his writings he consistently charged with being reductive or even misleading.

Bibliography

Unpublished

- HP stands for the Harrod Papers, Chiba University of Commerce, Ichikawa, Japan; JMP indicates the Marschak Papers, University of California, Los Angeles; LoN stands for League of Nations Library, Geneva. A stands for 'autograph', Cc for 'Carbon copy', I for 'initialed', L for 'letter', N for 'note', T for 'typed', S for 'signed'.
- Harrod to Haberler, 19 October 1934, TL, Cc transcription, in HP IV-395-422 and LoN 10B/12653/12653.
- Harrod to Haberler, 5 November 1934, TL, Cc transcription, in HP IV-395-422 and LoN 10B/12653/12653.
- Harrod to Marschak, 7 September 1938, ALS, in JMP 1275, Box 147, folder H.
- Marschak, Jacob 1938* 'Remarks on R.F.H.'s "Essay in Dynamic Theory"', ANI, undated, in HP V-113a.

Published

- Ackley, G. 1961. *Macroeconomic Theory*, New York, Macmillan
- Ahmad, S. 1961. Harrod on Domar's Theory of Growth, *Economic Journal*, vol. LXXI, 449-51
- Alexander, S. S. 1950. Mr. Harrod's Dynamic Model, *Economic Journal*, vol. LX, 724-39
- Allen, R. G. D. 1956. *Mathematical Economics*, London, Macmillan
- Anonymous 1937. [Review of] *The Trade Cycle*, by R. F. Harrod, *The New Statesman and Nations*, vol. 6, 220
- Asimakopulos, A. 1985. Harrod on Harrod: the evolution of 'a line of steady growth', *History of Political Economy*, vol. 17, no. 4, 619-35
- Asimakopulos, A. 1986. Harrod and Domar on dynamic economics, *Banca Nazionale del Lavoro Quarterly Review*, vol. 39, 275-98
- Baumol, W. J. 1948. Notes on some dynamic models, *Economic Journal*, vol. LVIII, 506-21
- Baumol, W. J. 1951. *Economic Dynamics. An Introduction*, New York, Macmillan
- Benham, F. 1938. [Review of] *The Trade Cycle*, by R. F. Harrod, *International Affairs*, January, 103-4
- Besomi, D. 1995. From *The Trade Cycle* to the 'Essay in dynamic theory'. The Harrod-Keynes correspondence, 1937-1938, *History of Political Economy*, vol. 27, no. 2, 309-43
- Besomi, D. 1996. Harrod's dynamics in 1938. An additional note on the Harrod-Keynes correspondence, *History of Political Economy*, vol. 28, no. 2, 281-94
- Besomi, D. 1997A. Roy Harrod and traditional theory, *European Journal for the History of Economic Thought*, vol. 4, no. 1, 92-115
- Besomi, D. 1997B. Statics and dynamics in Harrod's *Trade Cycle*, *Review of Political Economy*, vol. 9, no. 2, 181-209
- Brooman, F. S. 1962. *Macroeconomics*, London, Allen & Unwin
- Burmeister, E. and Dobell, A. R. 1970. *Mathematical Theories of Economic Growth*, New York, Macmillan

- Coleman, W. 1991. Harrod's Growth Model and the Multiplier–Accelerator Model: two sides of the same coin? *Metroeconomica*, vol. 42, no. 3, 199–212
- Costa, G. 1970. Considerazioni su alcuni concetti e problemi di Harrod, *Rivista Internazionale Scienze Sociali*, vol. 2, 393–407
- Di Matteo, M. 1987. *Warranted, Natural, and Actual Rates of Growth: Reflectons of a Perplex*, University of Siena, Quaderni dell'Istituto di Economia 68
- Dixit, A. K. 1976. *The Theory of Equilibrium Growth*, Oxford, Oxford University Press
- Dusenberry, J. S. 1958. *Business Cycles and Economic Growth*, New York, McGraw-Hill
- Fellner, W. 1951. The capital–output ratio in dynamic economics, pp. 105–34 in *Money, Trade, and Economic growth. In Honor of John Henry Williams*, New York, Macmillan
- Gaitskell, H. T. N. 1937. [Review of] *The Trade Cycle*, by R. F. Harrod, *Economica*, NS vol. IV, 472–6
- Glombowski, J. and Krüger, M. 1982. on instability principles in the context of growth cycle theory, *Economic Notes*, vol. 3, 130–47
- Goodwin, R. M. 1950. A nonlinear theory of the cycle, *Review of Economics and Statistics*, vol. XXXII, 316–20
- Goodwin, R. M. 1951. The nonlinear accelerator and the persistence of the business cycle, *Econometrica*, vol. 19, no. 1, 1–17
- Hacche, G. 1979. *The Theory of Economic Growth. An Introduction*, London, Macmillan
- Hahn, F. 1971. *Readings in the Theory of Growth*, London, Macmillan
- Hahn, F. and Matthews, R. C. O. 1964. The Theory of Economic Growth: a survey, *Economic Journal*, vol. LXXIV, 779–902
- Hamberg, D. 1956. *Economic Growth and Instability. A Study in the Problem of Capital Accumulation, Employment, and the Business Cycle*, New York, Norton
- Hamberg, D. 1971. *Models of Economic Growth*, New York, Harper & Row
- Harrod, R. F. 1927. Mr. Robertson's Views on Banking Policy, *Economica*, vol. XXXVII, 224–32
- Harrod, R. F. 1934A. Doctrines of Imperfect Competition, *Quarterly Journal of Economics*, vol. 48, 442–70
- Harrod, R. F. 1934B. The expansion of credit in an advancing community, *Economica*, NS vol. 1, 287–99
- Harrod, R. F. 1934C. Rejoinder to Mr. Robertson, *Economica*, NS vol. 1, 476–8
- Harrod, R. F. 1935. (Review of) *The Problem of Credit Policy*, by E. F. M. Durbin, *Economic Journal*, vol. XLV, 725–9
- Harrod, R. F. 1936. *The Trade Cycle. An Essay*, Oxford, Clarendon Press
- Harrod, R. F. 1937. [Review of] *Studies in the Theory of Economic Expansion* [by E. Lundberg], *Zeitschrift für Nationalökonomie*, vol. 8, no. 5, 494–8
- Harrod, R. F. 1938. Scope and method of economics, *Economic Journal*, vol. XLVIII, 383–412
- Harrod, R. F. 1939A. An essay in dynamic theory, *Economic Journal*, vol. XLIX, 14–33
- Harrod, R. F. 1939B. [Review of] *Value and Capital*, by J. R. Hicks, *Economic Journal*, vol. XLIX, 294–300
- Harrod, R. F. 1948. *Towards a Dynamic Economics*, London, Macmillan
- Harrod, R. F. 1951. Notes on trade cycle theory, *Economic Journal*, vol. XLI, 261–75
- Harrod, R. F. 1952A. *Economic Essays*, London, Macmillan
- Harrod, R. F. 1952B. [Review of] *The Dynamics of Business Cycles*, by Jan Tinbergen, *Economic Journal*, vol. LXII, 131–3
- Harrod, R. F. 1953. Comment [to Pilvin 1953], *Quarterly Journal of Economics*, vol. LXVII, no. 4, 553–9
- Harrod, R. F. 1955. Les Relations entre l'Investissement et la Population, *Revue Économique*, vol. 6, no. 3, 356–67
- Harrod, R. F. 1957A. [Review of] *Introduction to Keynesian Dynamics*, by K. K. Kurihara, *Annals of the American Academy*, vol. 313, 192–4
- Harrod, R. F. 1957B. Professor Fellner on Growth and Unemployment, *Kyklos*, vol. X, 1–17
- Harrod, R. F. 1959. Domar and dynamic economics, *Economic Journal*, vol. LXIX, 451–64
- Harrod, R. F. 1960. Second essay in dynamic theory, *Economic Journal*, vol. LXX, 277–93
- Harrod, R. F. 1962. A reply to Mr. Bilkey, *Economic Journal*, vol. LXXII, 1009–10
- Harrod, R. F. 1963. Themes in dynamic theory, *Economic Journal*, vol. VXXIII, 401–21
- Harrod, R. F. 1968. What is a model?, in Wolfe, J. N. (ed.), *Value, Capital, and Growth. Papers in Honour of Sir John Hicks*, Edinburgh, at the University Press

- Harrod, R. F. 1969. *Money*, London, Macmillan
- Harrod, R. F. 1970. Harrod after twenty-one years: a comment, *Economic Journal*, vol. LXXX, 737–41
- Harrod, R. F. 1971. *Sociology, Morals and Mystery*, London, Macmillan
- Harrod, R. F. 1973. *Economic Dynamics*, London, Macmillan
- Harrod, R. F. 1974. Pure theory of growth economics, *Zeitschrift für Nationalökonomie*, vol. 34, 241–7
- Harrod, R. F. 1976. Economic dynamics and economic policy, pp. 69–81 in Dopfer, K. (ed.), *Economics in the Future*, London, Macmillan
- Harrod, R. F. 1996. An essay in dynamic theory (1938 Draft), edited by D. Besomi, *History of Political Economy*, vol. 28, no. 2, 253–80
- Hicks, J. R. 1950. *A Contribution to the Theory of the Trade Cycle*, Oxford, Clarendon Press
- Ichimura, S. 1955. *The Historical Development of Economic Dynamics*, *Economic Series No. 4*, Tokyo, The Science Council of Japan,
- Jolink, A. 1995. 'Anecdotal Myths': Tinbergen's Influence on Harrod's Growth Theory, *European Journal of the History of Economic Thought*, vol. 2, no. 2, 434–49
- Jones, H. G. 1975. *An Introduction to Modern Theories of Economic Growth*, Sunbury-on-Thames, Nelson
- Kaldor, N. 1951. Mr. Hicks on the trade cycle, *Economic Journal*, vol. LXI, 833–47
- Kaldor, N. 1957. A model of economic growth, *Economic Journal*, vol. LXVII, 591–624
- Kalecki, M. 1962. Observations on the theory of growth, *Economic Journal*, vol. LXII, 134–53
- Keynes, J. M. 1973. *The Collected Writings of J. M. Keynes. Vol. XIV: The General Theory and After. II. Defence and Development*, edited by Moggridge, D., London, Macmillan
- Krauss, M. 1970. Cyclical movement on the Harrod model, *Osaka Economic Papers*, vol. 18, no. 1/2, 21–5
- Kregel, J. A. 1972. *The Theory of Economic Growth*, London, Macmillan
- Kregel, J. A. 1980. Economic dynamics and the theory of steady growth: an historical essay on Harrod's 'knife-edge', *History of Political Economy*, vol. 12, no. 1, 97–123
- Kregel, J. A. 1985. Harrod and Keynes: increasing returns, the theory of employment and dynamic economics, pp. 66–102 in Harcourt, G. C. (ed.), *Keynes and His Contemporaries*, London, Macmillan
- Lokanathan, P. S. 1938. Mr. Harrod on the trade cycle, *Indian Journal of Economics*, vol. XVIII, part III, 515–21
- McCord Wright, D. 1949. Mr. Harrod and growth economics, *Review of Economics and Statistics*, vol. XXXI, no. 4, 322–8
- Miconi, B. 1967. On Harrod's model and instability, *L'industria*, vol. 4, 455–78
- Nardozi, G. 1983. Crescita, pp. 113–219 in Lunghini, G. (ed.), *Dizionario di economia politica*, vol. 7, Torino, Boringhieri
- Neher, P. A. 1971. *Economic Growth and Development: A Mathematical Introduction*, New York, John Wiley
- Pasinetti, L. 1974. *Growth and Income Distribution. Essays in Economic Theory*, Cambridge, University Press
- Pilvin, H. 1952. A geometric analysis of recent growth models, *American Economic Review*, vol. XLII, no. 4, 594–9
- Pilvin, H. 1953. Full capacity vs. full employment growth, *Quarterly Journal of Economics*, vol. LXVII, no. 4, 545–53
- Predetti, A. 1961. *Il modello di R. F. Harrod*, Roma, Istituto Nazionale per lo Studio della Congiuntura
- Pugno, M. 1992. *Roy F. Harrod. Dall'equilibrio dinamico all'instabilità ciclica*, Bologna, Il Mulino
- Pugno, M. 1998. In search of Harrod's contribution to economic dynamics. A note, pp. 89–106 in Rampa, G., Stella, L. and Thirlwall, A. (eds), *Economic Dynamics, Trade and Growth: Essays on Harrodian Themes*, London, Macmillan
- Punzo, L. 1988. Harrodian macrodynamics in generalized coordinates, pp. 30–52 in Ricci, G. and Velupillai, K. (eds), *Growth Cycles and Multisectorial Economics: The Goodwin Tradition*, Berlin, Springer
- Robinson, J. V. 1936. [Review of] *The Trade Cycle* by R. F. Harrod, *Economic Journal*, vol. XLVI, 690–93, December
- Robinson, J. V. 1952. The model of an expanding economy, *Economic Journal*, vol. LXII, 42–53

- Robinson, J. V. 1970. Harrod after twenty-one years, *Economic Journal*, vol. LXXX, 731–7
- Rose, H. 1963. Expectations and stability in neo-Keynesian Growth Theory, *Quarterly Journal of Economics*, vol. LXXVII, no. 1, 71–94
- Sen, A. 1971. *Growth Economics*, Harmondsworth, Penguin
- Smithies, A. 1937. [Review of] *The Trade Cycle*, by R. F. Harrod, *Economic Record*, vol. 13, 109–11
- Solow, R. M. 1956. A contribution to the Theory of Economic Growth, *Quarterly Journal of Economics*, vol. LXX, no. 1, 65–94
- Stafford, J. 1937. The trade cycle, *The Manchester School*, vol. VIII, no. 1, 69–84
- Stiglitz, J. E. and Uzawa, H. 1969. *Readings in the Modern Theory of Economic Growth*, Cambridge, MA, and London, MIT Press
- Swan, T. W. 1956. Economic growth and capital accumulation, *Economic Record*, vol. 32, 334–61
- Tinbergen, J. 1937. (Review of) Harrod, R. F., *The Trade Cycle. An Essay*, *Weltwirtschaftliches Archiv*, vol. 45, no. 3, 89*–91*
- Wan, H. Y. 1971. *Economic Growth*, New York, Harcourt Brace
- Yeager, L. B. 1954. Some questions about growth economics, *American Economic Review*, vol. XLIV, no. 1, 53–63
- Young, W. 1989. *Harrod and his Trade Cycle Group. The Origins and Development of the Growth Research Programme*, London, Macmillan